Engineering Notes

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Rate of Climb for Light Propeller Powered Airplanes

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WHILE carrying out additional calculations for rates of climb of light airplanes from the equations given in Refs 1 and 2, two mistakes were discovered in our previous paper [2] A typing error had transposed the last two numbers given for τ in Fig 1 of Ref 2, so it should be changed to $\tau = 0\ 2278$ in order to provide at least three digit accuracy A more serious error was found in our calculation of $w = (dh/dh)_{max}$ from Eq (13) in Fig 2 of Ref 2 This curve was supposed to represent the maximum climb rate for a fixed pitch propeller that satisfied Oswald's[3] suggestion that the maximum available power should be absorbed at the maximum possible level flight velocity near sea level $(\sigma = 1 = \phi)$ Unfortunately, in the calculations from the correctly printed Eq (13) in Ref 2, the first factor was erroneously used as ⅔ (τ) A recalculation of w is shown in Fig 1, which replaces Fig 2 of Ref 2, and it is seen that the curve for w is now more nearly linear and is now properly below the curves for Eqs (14) and (15) which give the rate of climb $(\dot{h})$ for the ideal variable pitch propeller

In agreement with the data for most light airplanes the decrease in rate of climb w with altitude h is very linear a straight line from $(w)_{\sigma=1}$ *at* $h = 0$ to h_{abs} when $w = 0$ overestimates the rate of climb by less than 5% at $h/h_{abs} = ½$ and is given by

$$\frac{(dh/dt)_{max}}{(V_*)_{\sigma=1}} = \frac{w}{(\lambda_p \lambda_s)^{1/4}} = 0\ 1267\left(1 - \frac{h}{h_{abs}}\right) \qquad (1)$$

The absolute ceiling (h_{abs}) is determined from Eq (6) of Ref 2 by the value of $\sigma = \rho/\rho_0$ as

$$\sigma_h = C + \phi_h(1 - C) \quad \text{where } 0 \le C \le 0\ 22 \qquad (2)$$

The origin and the selection of the constant C is discussed in Ref 1 The classical time to climb for light airplanes with a negligible change in weight due to fuel consumption can be obtained by integrating Eq (1) so as to obtain (e g , see McCormick [4], pp 439 440):

$$t = \frac{h_{abs}}{w_{\sigma=1}} \ln\left(1 - \frac{h}{h_{abs}}\right)^{-1} \qquad (3)$$

From Eq (13) of Ref 2, or Fig 1 above, we have $w_{\sigma=1} = 4\ 841$ *m/s* $= 15\ 88$ ft/s $= 953$ fpm for the fixed pitch propeller defined in Ref 2 The so called ' service ceiling' is defined as the altitude (h_{ser}) where the rate of climb is reduced to $w_{ser} = 100$ fpm $= 0\ 508$ m/s For this Oswald[3] type fixed pitch propeller we can calculate from $\phi_h = 0\ 4585$ the absolute ceiling and the time (min) to climb to the service ceiling, both dependent upon the selected constant C in Eq (2) as

$$C = 0\ 15;\ \sigma_h = 0\ 5397;\ h_{abs} = 5980\text{m}(19\ 620\ \text{ft})$$
$$= h_{ser}/0\ 895;\ t = 46\ 4\ \text{min}$$
$$C = 0\ 21;\ \sigma_h = 0\ 5722;\ h_{abs} = 5450\text{m}(17{,}880\ \text{ft})$$
$$= h_{ser}/0\ 895;\ t = 42\ 3\ \text{min}$$

The constant C is a correction for the altitude effect upon $\phi\tau W$ in Ref 2 Quite often this term is simplified to $\sigma\tau W$ corresponding to $C = 0$; however as shown in Refs 1 and 2, this leads to an overestimation of the maximum velocity and the climb rate at the higher altitudes It is important to note that even though $\phi\tau W$ is always less than the static thrust at $V = 0$ still the basic assumption is that at any given altitude $\phi\tau W$ is a constant that must be independent of a slowly varying velocity at least in the range $V_* \le V \le V_{max}$ As long as this is true Eqs (7) (12) (13), and (16) of Ref 2 provide a good approximation for a fixed pitch propeller whose theoretical thrust variation has been shown to be given by Eq (3) of Ref 2, or Eq (10) of Ref 1 The correction term $\phi = (\sigma - c)/(1 - c)$ is only a crude approximation to the decrease in thrust with altitude A much better approximation to $\phi(h)$ can be obtained by plotting the calculated or ex perimental fixed pitch propeller thrust versus velocity and then using a "best fit' parabola in the range $V_* \le V \le V_{max}$ to approximate Eq (3) of Ref 2 It should be noted that the second term $-(\sigma/2)(P_m/WV_m)(V/V_m)^2$ comes directly from fixed pitch propeller theory where V_m is the velocity where the engine propeller combination produces its peak power (P_m) In any final detailed analysis the engine propeller combination must be considered as the best design compromise providing the desired power available (TV)

The curve representing Eq (13) of Ref 2 in Fig 1 is for $w = (dh/dt)_{max}$ for a climb with a fixed pitch propeller that has a sufficiently coarse (i e high) pitch angle so as to absorb the maximum available power at the maximum level flight speed near sea level $(\sigma = 1 = \phi)$ For the airplane considered in Ref 2 this corresponds to $V_m = V_{max} = 69\ 54$ m/s $= 228\ 15$ ft/s $\tau = 0\ 2278$ and $\nu^{-1} = 1\ 57 \times 10^{-5}$ (s/m)^2 $= 1\ 459 \times 10^6$ (s/ft)^2 As shown in Refs 1 and 2 the rate of climb near sea level can be approximately doubled if a fixed pitch propeller is selected with a sufficiently fine (i e low) pitch angle so that at $\sigma = 1 = \phi$ the maximum available power is absorbed at $V_m = V_* = 38\ 2$ m/s $= 125\ 3$ ft/s For this case Eq (4) of Ref 2 yields $\tau_* = 0\ 4147$ and $\nu_*^{-1} = 9\ 471 \times 10^{-5}$ (s/m)^2 $= 8\ 799 \times 10^{-6}$ (s/ft)^2 so that the sea level rate of climb from Eq (13) of Ref 2 becomes $w = 7\ 47$ m/s $= 24\ 51$ ft/s $= 1470$ fpm at a trim speed of $V_w = 35\ 4$ m/s $= 116\ 1$ ft/s This corresponds to a trim lift coefficient of $C_{L_w} = 0\ 833$ at $\sigma = 1 = \phi$ which is 17% greater than the lift coefficient for minimum drag $C_{L_*} = 0\ 714$ so that this airplane's climb is in the unstable speed range As shown in Houghton and Carruthers[5] any flight speed less than that for minimum drag (V_*) is inherently speed unstable and requires constant corrections

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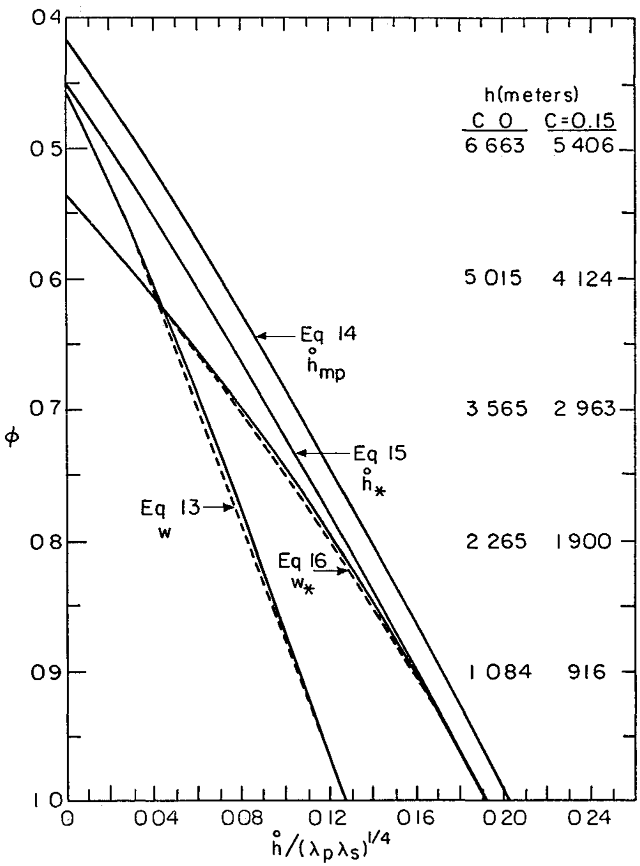

Fig 1 Corrected Fig 2 of Ref 2 Curve for w from Eq (13) of Ref 2 for a fixed pitch propeller with $V_m = V_{max}$ was incorrect The solid lines are for $C=0$ and the dotted lines are for $C=0$ 15 $(\lambda_p \lambda_s)^{1/4} = (V_*)_{\sigma=1} = 38$ 2 m/s = 125 3 ft/s

by the pilot because any speed reduction now results in an increase in drag which slows the aiplane even further In addition this finer pitch propeller having $V_m = V_*$ for Eq (13) predicts an absolute ceiling from Eqs (5) and (6) of Ref 2 as $\phi_h = 0$ 42; $V_h = 41$ m/s = 134 5 ft/s; and $C_{L_h} = 1$ 48 which is obviously impossible Therefore the proposed operation for this finer pitch propeller was given by Eq (16) of Ref 2 and is shown above as well as in Fig. 2 of Ref 2 as one which restricts the climb to $C_L = 0$ 7141 at a trim speed of V_* now corresponding to the maximum lift to drag ratio This finer pitch propeller should provide a quicker takeoff run since $\tau_* = 0$ 4147 = 1 82 τ where $\tau = 0$ 2278 for the coarser pitch propeller having $V_m = V_{max}$ for peak power Although the first term τW in Eq (3) of Ref 2 is always less than the actual static thrust the above calculations indicate the static thrust should be considerably greater for the finer pitch propeller; of course its maximum level flight speed must be less From Eq (7) of Ref 2 we calculate $V_{max} = 56$ 6 m/s = 185 7 ft/s near sea level $(\sigma = 1 = \phi)$ which is 23% less than that of the coarser pitch propeller The altitude prediction from Eq (16) is also more reasonable than that from Eq (13) for this finer pitch propeller since $C_{L_h} = C_L = 0$ 7141 is independent of C and Eq (6) of Ref 2 gives a reasonable value of $\phi_h = 0$ 5359 $= (\sigma_h - c)/(1-c)$ so that for $C = 0$ 073 we have $\sigma_h = 0$ 57 and $h_{abs} = 5$ 500 m = 18 000 ft

Finally, it should be noted that although Eq (13) of Ref 2 does provide a good approximation for the coarser pitch Oswald[3] type propeller still for the higher altitudes Fig 1 shows that for $\phi < 0$ 62 $(h/h_{abs} > 0$ 65$) w > w_*$ since $C_{L_w} > C_L$; still C_{L_w} does not become too large for the drag polar approximation since $C_{L_h} = 0$ 888 < 1 is independent of C However the climb at these higher altitudes requires constant pilot attention because of the inherent speed drag instability previously mentioned Likewise the climb with a variable pitch propeller is more stable and more likely to be attained by Eq (15) in Ref 2 (see Fig 2 of Ref 2 or Fig 1 above) with climb at a constant C_{L*} rather than by Eq (14) with climb at a constant $C_{l_{mp}} = \sqrt{3} C_L = 1$ 24 > 1 Not only is the latter climb uncomfortably speed drag unstable but for this air plane the predicted rate of climb cannot be attained since as shown in Fig. 1 of Ref 1 the actual drag is 10% greater than that predicted by the drag polar approximation because $C_{l_{mp}} = 1$ 24 > 1

The final conclusion of our calculations is that one should climb or glide at a trim speed slightly greater than V_* in any conventional light airplane as long as $C_L < 1$

References

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